- 7 -
CROSSOVERS

ELECTRICAL AND ACOUSTICAL ASPECTS

7.1 Background

Crossover networks are an integral part of any loudspeaker system. A loudspeaker system is a combination of individual drivers designed to achieve the objective of wide bandwidth. The need for a system of drivers is because it is unusual to find a single driver which can meet all of the goals of hi-fidelity sound reproduction, although there have been attempts that worked reasonably well. In most cases two or more drivers are required to meet the bandwidth objectives while achieving reasonably high output, it being a much easier task to obtain high bandwidth at lower output levels.

The fundamental problem that one has to deal with when transferring the sound output from one driver to the next is the fact that they generally do not occupy the same location in space. Since no crossover can transition instantaneously, there will always be an overlap region where the two drivers are both radiating sound. If they are not at the same physical location in space (more accurately if the acoustic centers are not at the same point), then interference patterns will be evident in the polar response. It is our task in this chapter to discuss the fundamental characteristics of this problem, not to do an exhaustive study of the problem.

It is not uncommon in loudspeaker systems designs to use the components of the crossover to help to "correct" the response of the driver to which it is attached. We will not delve into this aspect of the subject, only because to elaborate on all aspects of these techniques would be more of an electronics problem than an acoustics one, acoustics being the central subject of this text. In all passive circuits, there is always coupling between the passive elements and the transducers electroacoustic elements. For this reason, we will discuss only the fundamentals in order to give the reader some background in the techniques used and a familiarity with the problems. We will not be going into much detail.

The real task in this chapter is to show that the crossover is trying to achieve an acoustical objective and not an electrical one. The goal of the crossover is a net acoustical response. One cannot design a crossover independent of the transducers involved. The discussion in this chapter will serve to define techniques which can be used to achieve the desired acoustical response.

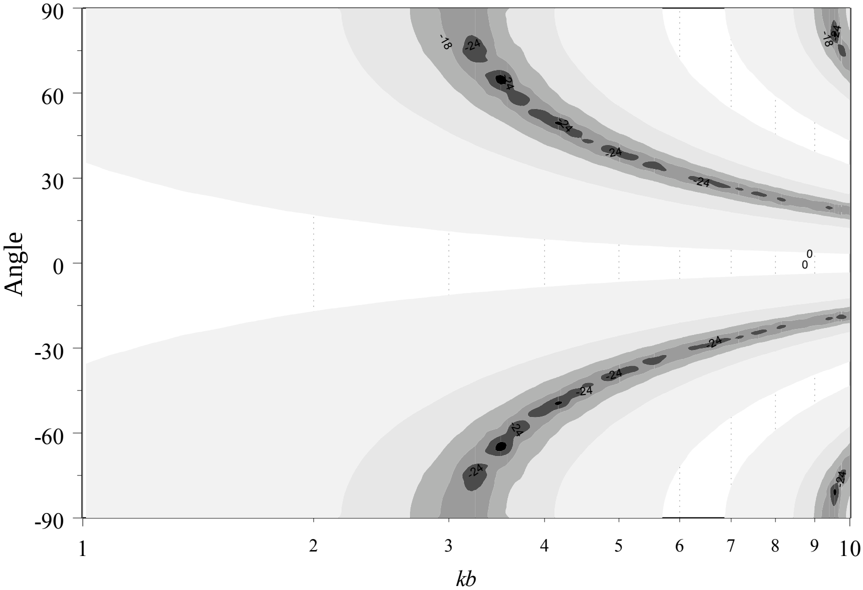

Figure 7-1 - A polar map of a bipole with separation b

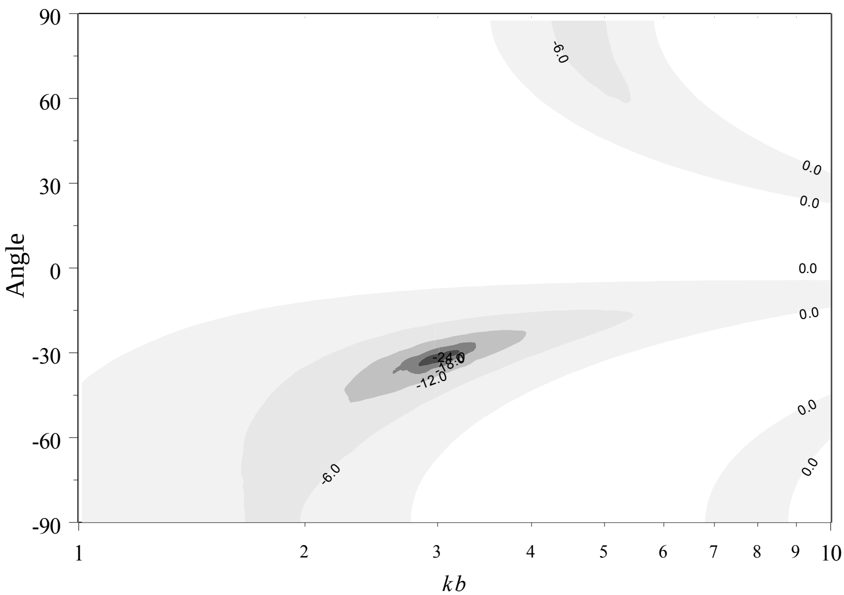

Figure 7-2 - First order filters with point sources.

7.2 Two-Way Crossovers

Fig. 7-1 shows the response of two identical drivers spaced apart by a fixed distance *b*. (Throughout most of this chapter we will plot the response as a function of *kb* – a dimensionless number which makes the curves applicable to any situation.) We can see from this plot that the two drivers interfere with one another creating null lines in this space which converge toward higher frequencies. The light areas are the polar lobes which get narrower and closer together as the frequency goes up (note that the finite resolution of the plotting routine does not make these nulls lines appear as continuous lines, although they really are). Throughout this chapter, we will show polar maps only in the plane containing the two sources. This is because the plane perpendicular to a line connecting the two sources has no aberrations that result from the crossover.

Consider now the same two sources with a crossover attached such that one source fades in as the other fades out. We will start with a first order filter for simplicity. The crossover point is at $kb = 3.0$. The response is shown in Fig. 7-2. There are several things to note from this plot. First, that the polar response is no longer symmetric. Second, that there is a null that is now finite in frequency and lowest at the crossover point. If we reverse the phase of the two drivers the plot simply changes to its reverse in θ, i.e. a vertical mirror image.

Moving on to second order filters we can see from Fig. 7-3 that a serious problem is starting to emerge. There is a large hole (complete sound cancellation) occurs in the response directly on axis centered at the crossover frequency. By simply reversing the phase of one of the drivers, we can move this hole off of the axis, which is a common technique. The point to notice here is that this filter combination is once again symmetric in theta and that there is always a hole – somewhere – as there must be.

The first three figures are idealistic in that they were calculated using point sources. The real situation is more complex in that the drivers have both amplitude and phase characteristics as well as directivity of there own. These effects have a considerable effect on the net response, but the characteristic of a hole somewhere in the polar map will remain a constant. There are an infinite number of variations that one could have once we start to consider the other acoustic characteristics. This will limit us to only showing a few with, hopefully, the most important effects taken into account.

The plot in Fig. 7-4 shows a second order crossover with the two dominant acoustical effects considered. The first effect is the finite directivity of the low frequency (LF) unit and the second is the high pass magnitude and phase variations of the high frequency (HF) driver due to its resonance. The directivity of the HF driver is negligible in this example since it is small and the polar response is unity and the high pass function of the LF driver is below the region being examined. The HF drivers resonance is set to about ½ octave below the crossover frequency. The LF driver has a radius of one half the spacing between the two driv-

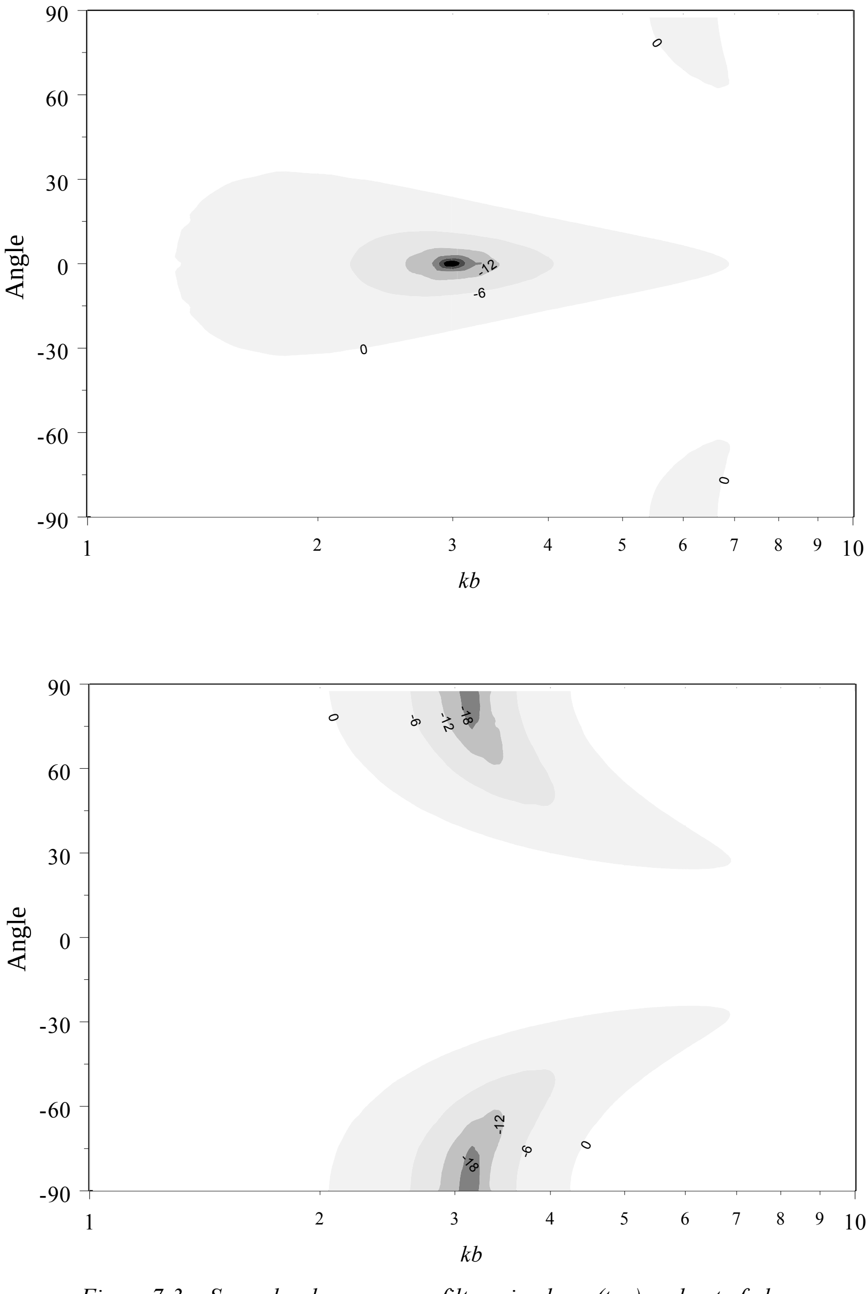

Figure 7-3 - Second order crossover filters, in phase (top) and out of phase

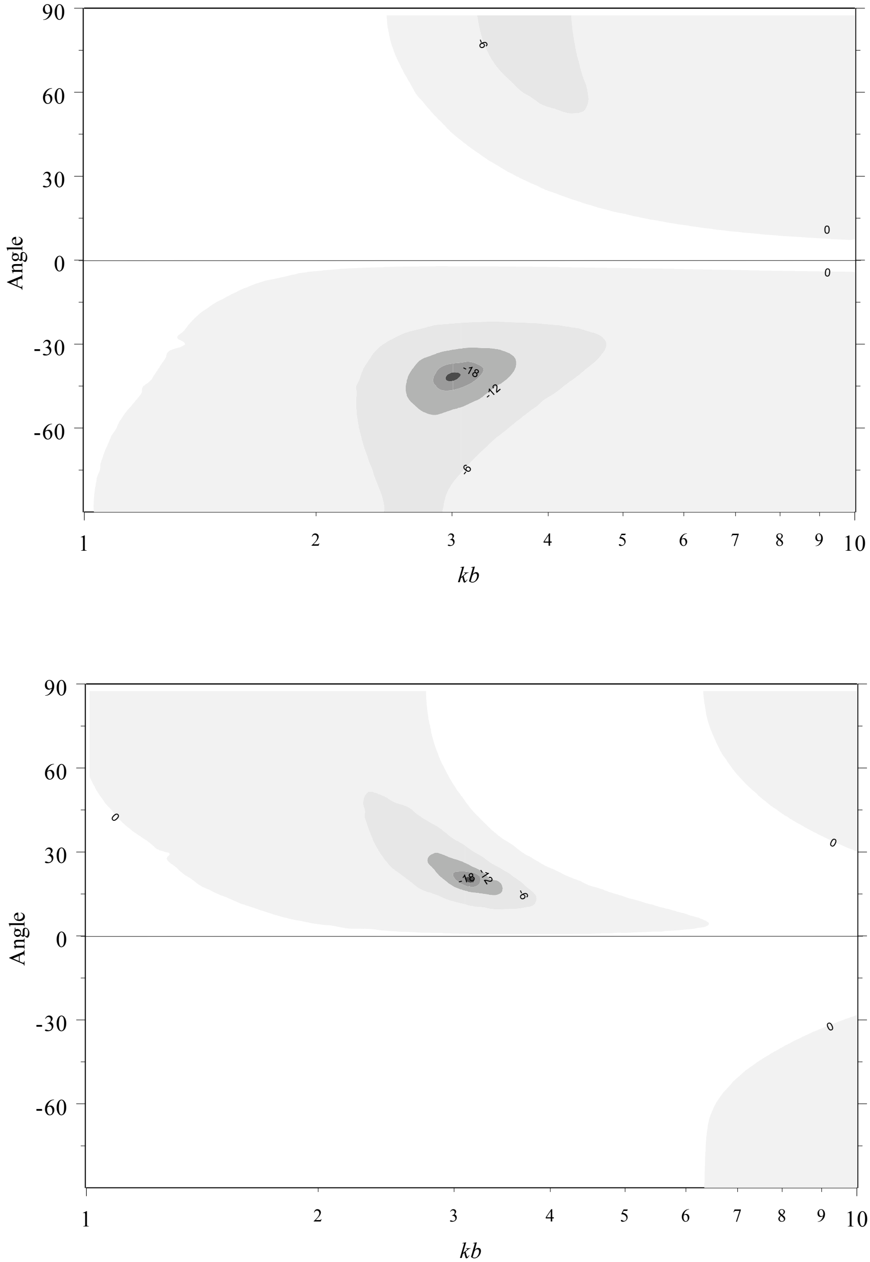

Figure 7-4 - Second order crossover, in phase (top) and out of phase, with driver directivity and resonance amplitude and phase considered ers and the two drivers are wired out of phase in the top graph and in-phase in the bottom graph.

This example shows that while the hole is no longer directly on axis it has not moved far off the central axis and its effect can still be seen along the axial direction. By switching the phase of the filter back to its normal polarity we get the response shown on the bottom of Fig. 7-4. Clearly we can see that there is no fixed rule about the phase of the filter as each instance must be examined as a unique situation. There is a sensitivity in the location of the response hole to small changes in the drivers parameters because the phase, which is the dominant factor in the location of this inevitable hole, is very sensitive to small changes in the drivers as well as the total design. Factors which have a negligible effect on the magnitude response of any one source can have an enormous effect on the system response.

The next logical discussion would be to consider a third order filter. Plots of these responses will not be shown since they are completely predictable. A hole always exists at the crossover frequency for point sources. It is smaller in "size" (as seen in a polar map) owing to the fact that it has a faster phase transition (more group delay). For in-phase point sources, the hole is on axis and for out of phase sources it moves to the 90° point. The location is slightly dependent on the filter coefficients, but not dominantly so.

The response of a third order filter with real sources is not so different from the response shown in Fig. 7-4 except that there are two holes in the in phase plot near the to 90° lines. The hole is on-axis when the sources are wired out of phase.

We can see that in general, to a first order, the crossovers behave quite predictably, even when real drivers are considered. There does not seem to be any magic combination that makes the problems go away.

7.3 Examples

In order to show some examples of how the crossover affects the actual design of a loudspeaker system we will only consider two examples out of the myriad of possibilities. Our first example will be a simulation of a common two way design with a 10 cm radius LF driver and a 1.2 cm HF driver. The drivers are spaced apart by 14 cm, and the crossover is chosen to be typical at about 2 kHz. The electrical filters are a first order high pass (HP) set at 3 kHz for the HF driver and 1.2 kHz second order low pass (LP) filter for the LF driver. As a general rule, loudspeakers require less overlap than electrical filters for constant summing of their axial outputs. The crossover is wired in-phase.

Fig. 7-5 shows the frequency responses for the individual drivers as well as the summed axial response. There is a small (less than 1 dB) ripple in the axial response. Reversing the polarity on one of the drivers results in a large hole in the axial response, so this phasing is not an option. The polar map is shown in Fig. 7-6. The vertical polar response has only a small narrowing at the crossover point. If the goal is to create as wide a directivity as possible then this design is quite

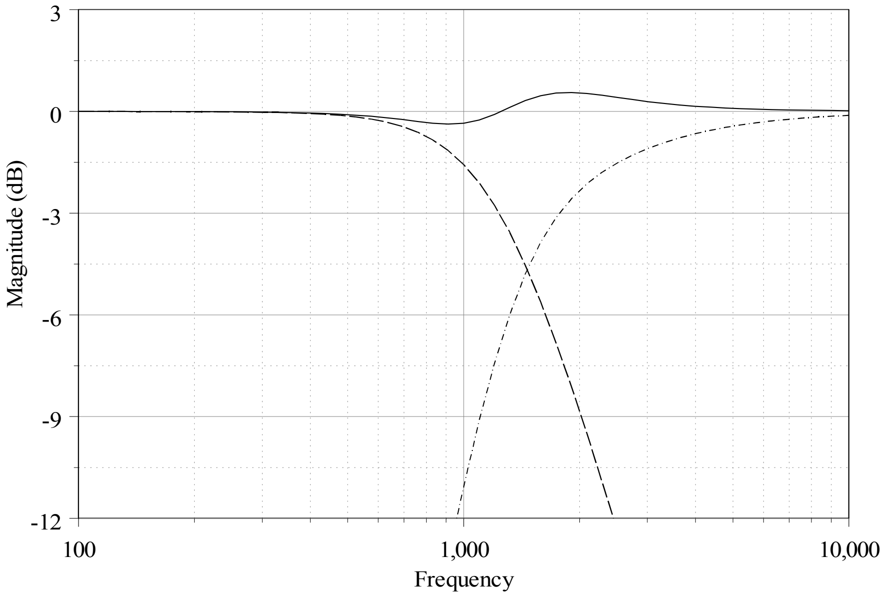

Figure 7-5 - Axial frequency response for typical two way loudspeaker

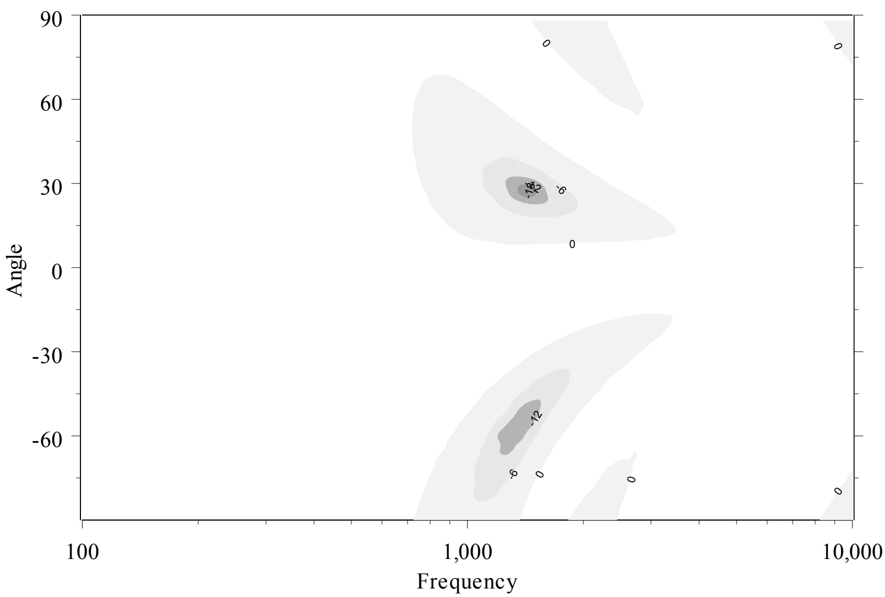

Figure 7-6 - Polar response maps for a typical two way loudspeaker system good. There is a reduction in the power response at about 1.5 kHz, but otherwise the power response is quite smooth. The power response is easily seen from the polar map. It is simply the integral along each vertical line, easily done by eye. When there is no variation in these integrations (averages) then the power response will be flat with frequency.

As we will see in the chapter on room acoustics, there are instances when one does not want a wide directivity, but a narrower directivity. Highly directive sound reduces the negative effects of small rooms and is used to control the sound distribution in larger rooms.

Lets now consider two drivers, one LF, and one higher, but probably not the highest frequency device. The LF device has a diameter of 40 cm and the other driver a diameter of 20 cm, fairly large devices. They are separated by 65 cm. The polar maps for this configuration are shown in Fig. 7-7. The crossover is made up of a single first order HP filter (a capacitor in series with the driver) and a second order LP filter. When coupled to the second order HP function of the mid/high frequency unit it creates an acoustic third order filter of fairly steep slope (see Fig. 7-8). Once again the crossovers filters do not overlap at the -3 dB point as electrical filters with constant power transfer would. They are about -5 dB at the crossing point.

We can see that the directivity of this system is considerably narrower than our previous example. In fact, it is probably too narrow at the upper end. What is apparent, however, is that this arrangement sends a great deal more of the sound

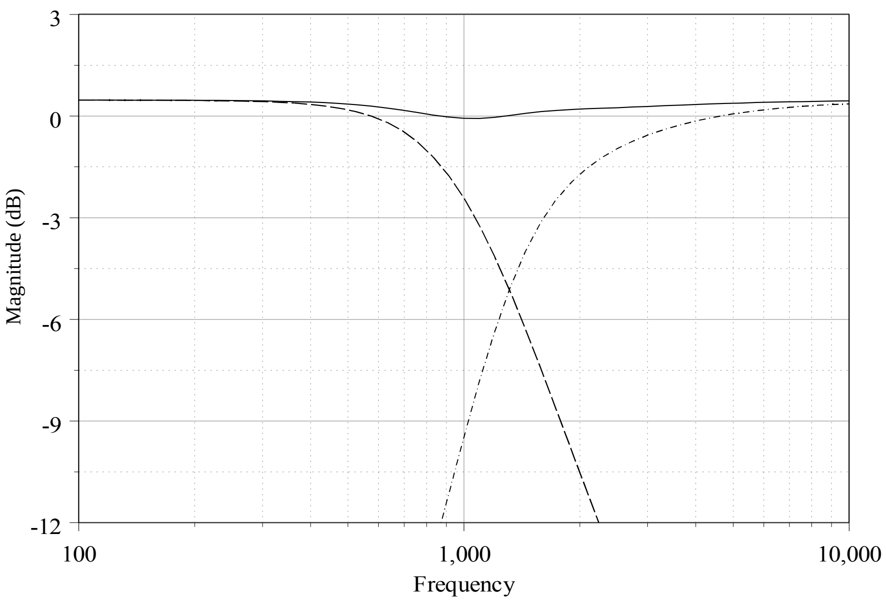

Figure 7-8 - The axial response of the large driver response

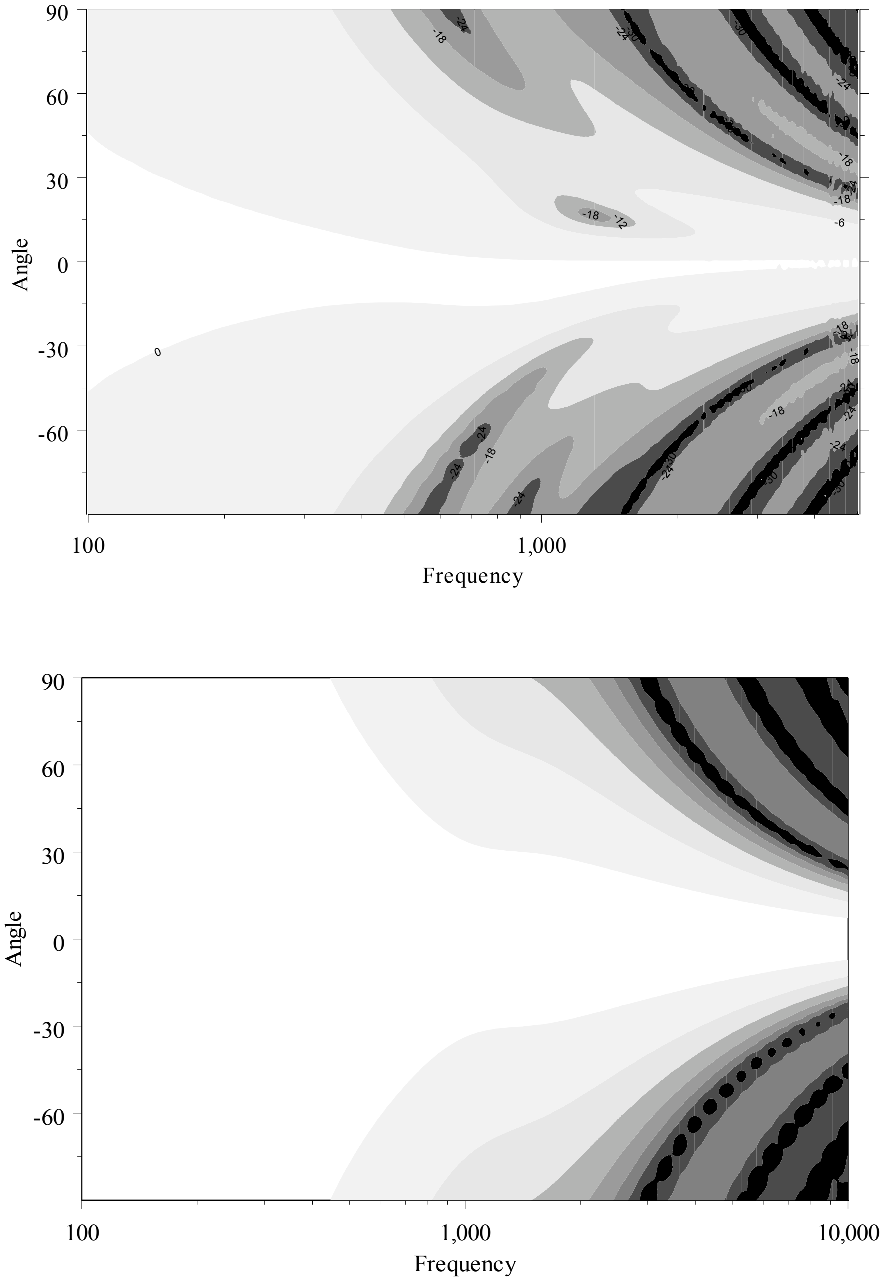

Figure 7-7 - Vertical and horizontal polar maps for directional two way system energy into the forward direction (less energy into the room boundaries) and appears to have reasonable directivity control from about 500 Hz for the vertical and 1 kHz for the horizontal up to about 5 kHz, where it is becoming quite narrow. This example would need to have a third driver, added at about 5 kHz, with a fairly narrow directivity, in order to approach a constant directivity system in the audio band (at least above 500 Hz). It would be desirable to be able to control the directivity to a lower frequency, but this cannot be done with typical transducer sizes available today, unless we array them. The entire subject of using multiple drive units or enclosure augmentations to perform directivity tasks which cannot be done by a single driver is the subject of a Chap. 9. We will pick up this discussion again at that point.

The point that we are making with this example is that the crossover can be used in conjunction with the driver's directivity to help to create a more narrow directivity with reasonable control over frequency than is possible with a driver alone. As we know the crossover always has response holes, and here we can see that these can be used to help us to control the directivity response. This aspect of the crossover design, namely directivity control, is an extremely important consideration that is often overlooked.

7.4 Passive Electrical Effects

Up until now, we have considered only crossovers which are not affected by the input impedance of the drivers. This is accurate for active (electronic) filters, but not for passive ones. When the crossover is implemented passively the drivers impedance must be accounted for. We have saw an example of this in Chap. 1. We will look at some further examples here.

When the crossover filter is LP, then the filter frequency tends to be well above the drivers resonance frequency and the only substantial impedance effect seen by the filter is the reactive impedance of the motors inductance.

Fig. 7-9 shows the voltage seen at the transducers input terminals for the case of a constant load and a more realistic transducer load. The curve is in terms of the normalized frequency where, $\omega = 1$ is the crossover point. This large inductance is required because of the normalized frequency range. The filter is a series inductance with a parallel capacitance with values of 10 H and .1 F, respectively, (again large values because of the normalized frequency) for the constant load curve.

The driver's impedance is not a real problem in the LP case since by simply adjusting the inductance and the capacitance values we can always achieve an acceptable response. The transducer has a real part of 8Ω and an inductance of 8 H. The second curve in Fig. 7-9 has new values of the inductor and capacitor of 16 H and .17 F. The transducers inductance actually helps create a steeper falloff of the response with frequency.

The case of a HP filter is substantially more complex that the LP case. In the HP case, one is often near the resonance of the driver and there will be a pro-

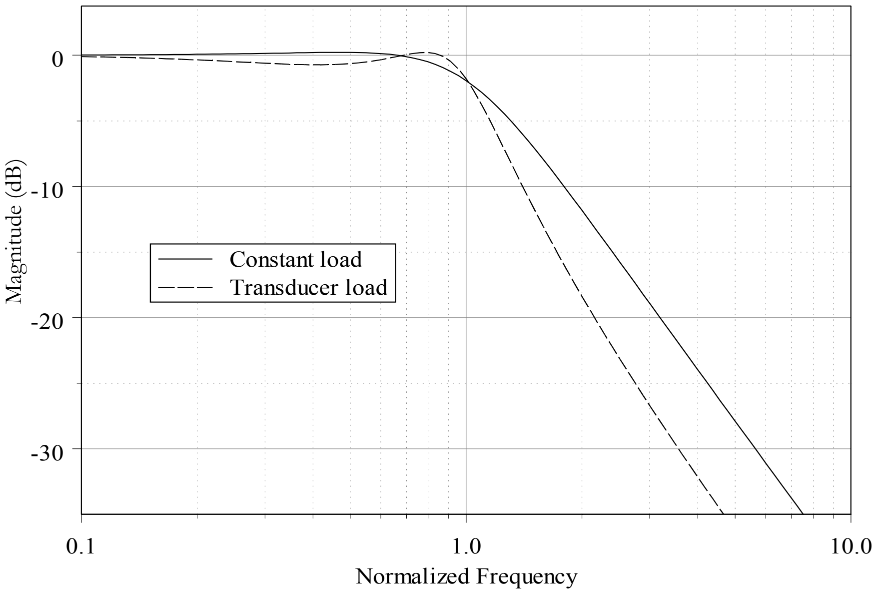

Figure 7-9 - The electrical response of a passive low pass filter loaded by a transducer impedance

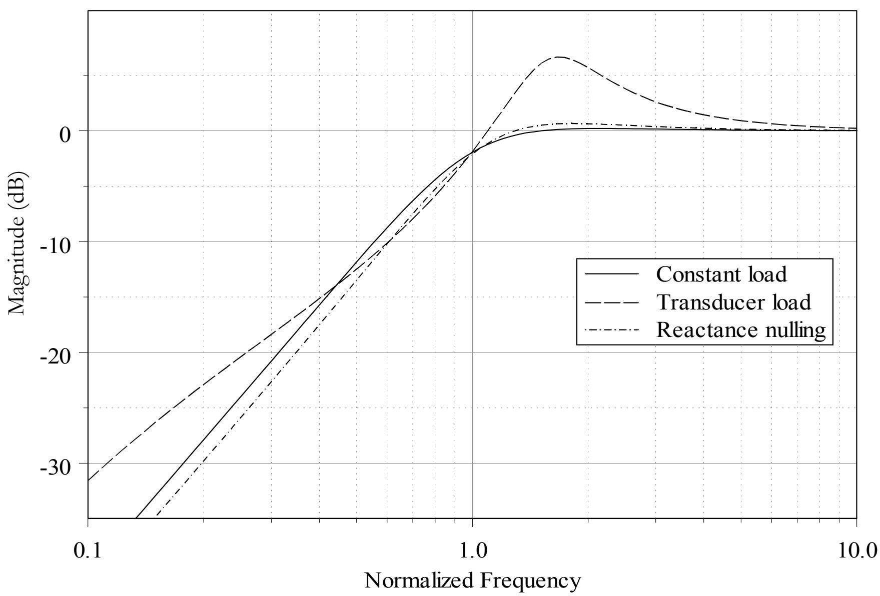

Figure 7-10 - The electrical response of a passive high pass filter with a constant load, a transducer load and a nulling circuit nounced effect on the total response of the passive filter resulting from the resonance peak in the transducers impedance. We will first consider the simpler case, where we are substantially above the driver's resonance so that the dominant effect will be that of the series inductance. The electrical response of a HP filter with a constant load, as well as a load with both a resistance and an inductance, are shown in Fig. 7-10. The HP function with a transducer load does not appear to be adequate. This is because the drivers inductance tends to null out the desirable effect of the passive inductance, creating an approximately first order section controlled by the capacitor alone. There is also an undesirable hump in the response which we will want to get rid of.

There are ways to modify the filter so that we get the response that we desire. Consider the T-matrix form for the filter, driver and an inserted section, which corresponds to a circuit of unknown impedance z_n in parallel to the driver. (Twenty-twenty hindsight tells us where to place this impedance.) We will have

$$\begin{pmatrix} E \\ I \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \dfrac{1}{-i\omega L} & 1 \end{pmatrix} \begin{pmatrix} 1 & \dfrac{1}{-i\omega C} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \dfrac{1}{z_n} & 1 \end{pmatrix} \begin{pmatrix} E_t \\ I_t \end{pmatrix} \tag{7.4.1}$$

L = passive filter parallel inductance
C = passive filter series capacitance
z_n = unknown parallel impedance required to null out the driver reactance
E = input voltage
I = input current
E_t = voltage seen at the transducer input
I_t = current into the transducer

Remember that the driver's current is not independent of the voltage. It is fixed by the drivers input impedance such that:

$$I_t = \frac{E_t}{z_t} \tag{7.4.2}$$

z_t = the transducer's electrical input impedance

Multiply out just the last matrix and the final vector in Eq. (7.4.1) and use Eq. (7.4.2) for I_t. We will get

$$\begin{pmatrix} 1 \\ \dfrac{1}{z_n} + \dfrac{1}{z_t} \end{pmatrix} E_t \tag{7.4.3}$$

In order to form a constant load for the electrical filter, the above terms must be equal to

$$\begin{pmatrix} 1 \\ \dfrac{1}{R_d} \end{pmatrix} E_t \tag{7.4.4}$$

R_d = the desired load resistance seen by the passive filter

Setting the terms in the above equations equal leads to the equation

$$z_n = \frac{1}{\dfrac{1}{R_d} - \dfrac{1}{z_t}} \tag{7.4.5}$$

We could require any value of resistance (even a complex impedance), but small values would simply be a waste of power. It is most effective to set $R_d = R_e$, the electrical resistance of the transducer. For this example we will use only the high frequency impedance of the transducer (we could use the complex one but that would just be more algebra)

$$z_t = R_e - i\omega L_e \tag{7.4.6}$$

L_e = electrical inductance of the transducer

By inserting Eq. (7.4.6) into Eq. (7.4.5), we can solve for the required impedance

$$z_n = R_e + \frac{1}{-i\omega \dfrac{L_e}{R_e^2}} \tag{7.4.7}$$

which we can see as a resistor R_e in series with a capacitor of value L_e / R_e^2.

The last curve in Fig. 7-10 has the above impedance placed across the transducer. As we can see, the modified passive filter works almost exactly like the passive filter loaded by a constant resistance.

We will now consider the case where the crossover point of the passive electric HP filter is close to the driver's resonance frequency. There are two factors to consider. First, the load impedance presented to the filter due to the driver is fairly complex. We know that we could null out this impedance with a complex circuit network, but this does not compensate for the second effect, which is the acoustic response. The HP acoustic response of a loudspeaker must be considered in the design of a HP crossover section whenever the two are close together. The most effective way to do this is to consider the driver and its passive electric filter to be a single HP filter in mixed domains.

Using the standard form for a transducer, we add a series capacitor

$$\begin{pmatrix} E(\omega) \\ I(\omega) \end{pmatrix} = \begin{pmatrix} 1 & \dfrac{1}{-i\omega C} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & R_e \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & Bl \\ Bl & 0 \end{pmatrix} \begin{pmatrix} 1 & z_m \\ 0 & 1 \end{pmatrix} \begin{pmatrix} F(\omega) \\ v(\omega) \end{pmatrix} \tag{7.4.8}$$

where all of these variables are known from previous equations. Multiplying these matrices out, ignoring terms in the radiation impedance and substituting for the mechanical impedance we will get

$$p(\omega) \approx \frac{-i\omega^3 Bl C_m C}{i\omega^3 M C_m R_e C - \omega^2\left(M C_m + Bl^2 C_m C + R_m C_m R_e C\right) - i\omega\left(R_m C_m + R_e C\right) + 1} \qquad (7.4.9)$$

$M = mechanical\ mass$

$C_m = mechanical\ compliance$

$R_m = mechanical\ resistance$

$Bl = voice\ coil\ coupling\ factor$

$C = series\ filter\ capacitor$

which can clearly be seen to be a third order HP filter with coefficients that are mixed mechanical and electrical quantities. It is also apparent that this filter is not simply the combination of an first order HP electrical filter and an second order HP acoustical filter since the function cannot be factored as such. It should also be clear that we cannot necessarily make a given HP function with a given loudspeaker. There are three coefficients that must be fit in order to yield an arbitrary filter function. We need three variables in order to do this. One is always the series capacitor, but the other two must be parameters of the loudspeaker itself. While not obvious, it is true that at least one of these three variables must be a mechanical quantity.

As an example, consider the design of a Butterworth HP filter for the following midrange speaker.

$C_m = 7.0 \cdot 10^{-5}\ m./Nt.$

$R_m = 3.0\ Mech.\ \Omega$

$R_e = 8.0\ \Omega$

$M = TBD$

$Bl = TBD$

$C = TBD$

Then the three equations become

$$M\,C = \frac{1}{R_e\, C_m\, \omega_0^3} = 1.79 * 10^{-6}$$

$$\left(Bl^2 + R_m R_e\right)C + M = \frac{2}{C_m\, \omega_0^2} \Rightarrow \left(Bl^2 + 24.0\right)C + M = 2.86 * 10^{-2}$$

$$C = \frac{\dfrac{2}{\omega_0} - R_m C_m}{R_e} = 224\ uF$$

$$M = 8.0 grams$$

$$Bl = 8.24 Nt./Amp$$

where $\omega_0 = 1000\,s^{-1}$. The response of this filter is shown in Fig. 7-11 for the above example.

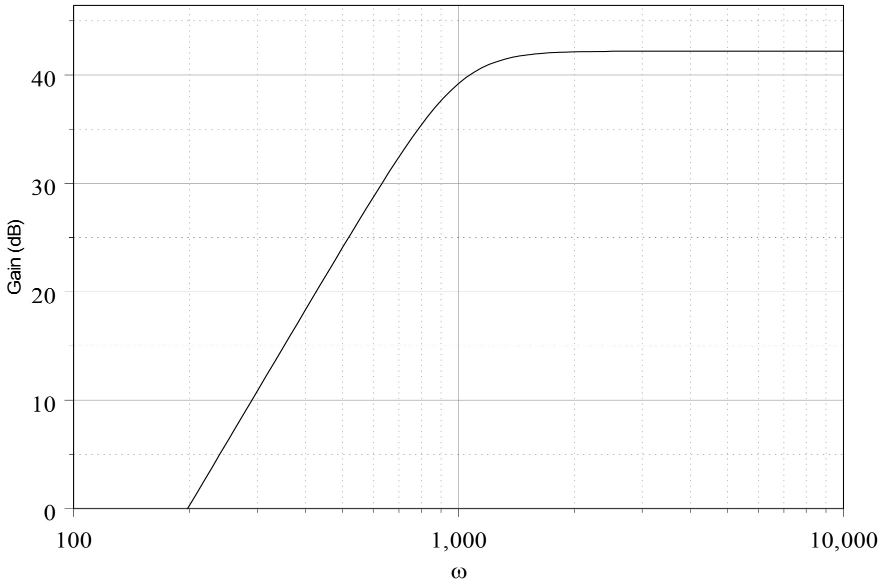

Figure 7-11 - Third order electroacoustic HP filter using a single capacitor

7.5 Acoustic Filters As Crossover Components

The last topic that we will talk about in this chapter is the concept of using an acoustic filter as part of the crossover network. Since what we want is a final crossover filter response in the acoustic domain it is quite reasonable to consider using an acoustic filter to help perform this task. To our knowledge, this has not been used in practice.

Consider the T-matrix for a front enclosed ported enclosure. We will use a duct in this port, although a passive radiator would also work. The matrix is

$$\begin{pmatrix} F \\ v \end{pmatrix} = \begin{pmatrix} S_d & 0 \\ 0 & S_d^{-1} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -i\omega \dfrac{V}{\rho c^2} & 1 \end{pmatrix} \begin{pmatrix} 1 & -i\omega \dfrac{\rho L}{A_d} + R_{\text{int}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} P \\ U \end{pmatrix} \tag{7.5.10}$$

V = volume in front of transducer
L = length of duct
A_d = area of duct
R_{int} = internal resistance added to duct

The other quantities are all know. If we multiply out this equation and derive the transfer function from the diaphragm volume velocity to the radiated volume velocity we will get

$$U(\omega) = \frac{v(\omega)\, S_d}{-\left(\dfrac{k}{k_0}\right)^2 - i\,\dfrac{k}{k_0}\sqrt{\dfrac{A_d}{V\,L}}\left[\dfrac{z}{\rho c}\left(A_d L + V\right) + \dfrac{V}{\rho c} R_{\text{int}}\right] + 1} \tag{7.5.11}$$

$$k_0 = \sqrt{\frac{A_d}{V\ L}}$$

where we have written the equation in k because it is simpler. We can recognize this as a standard second order lowpass filter section with a cutoff k_0 and a Q of

$$Q = \sqrt{\frac{V\,L}{A_d}}\,\frac{\rho c}{z\left(A_d L + V\right) + V\,R_{\text{int}}} \tag{7.5.12}$$

If there is no R_{int}, then the filter is damped only by the radiation impedance. If we assume that the radiation impedance is approximately $\rho c/A_d$, which it would be at higher frequencies, then the Q value simplifies to

$$Q = \frac{\sqrt{A_d\,L\,V}}{V + A_d\,L} \tag{7.5.13}$$

which are strictly variables in the acoustic filter. In general, to achieve Q's of about one, the duct length L would have to be very small to maintain the desired cutoff and would not be easy to achieve, albeit doable. It would be more practical to have some value of R_{int} to control the Q of this filter. This can be achieved quite simply with the use of a grill cloth as a damping mechanism. The final filter shape will be the product of an electrical filter and this acoustic one – properly designed for the transducer's load.

We would like to point out, that if one uses an acoustic lowpass filter as a crossover component that the distortion received in the field is lowered by the lowpass filter effect, which is not true for an equivalent electrical filter. The acoustic filter could also be far more cost effective to achieve.

7.6 Summary

The reader will, by now, have noted a substantial lack of analytical formula's and theoretical discussion of crossovers in this chapter. That is because there really are no theories in this subject to develop. There are, of course, a multitude of equations, and filter names in the electronics literature. However they really don't apply to our problem. We have seen in each of our examples that simple electrical theories simply do not apply. These electrical filters are scalar in nature and the acoustics problem is three dimensional. What one wants as an end result is an acoustic filter response which take into consideration the transducers amplitude and phase, neither of which can be ignored, as we have seen.

Crossover filters must be matched to the specific set of drivers being used and there really are few generalizations that one could make, other than to conclude that simply using electrical filter concepts does not work. We have shown how to accommodate the actual transducers and or enclosure response into the filter function and shown examples of such. We have not, nor will we, exhaust the myriad of application examples that could have been shown.