Weighting Up

By Keith Howard

For over 50 years it has been known that THD is a poor measure of perceived distortion. For just as long, weighting schemes have been proposed which improve the correlation between objective and subjective assessment, but these have been widely ignored. Now a new distortion metric has been developed that builds on these earlier attempts. Will it, ponders this article's author, prove any more influential?

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Sound reproduction systems are prone to two different forms of distortion: linear distortions, in which the amplitude versus frequency, phase versus frequency, or time behavior of the signal are altered, and nonlinear distortions, where additional, unwanted frequency components are added. Traditionally, as every audiophile knows, nonlinear distortion has been quantified using measurements of total harmonic distortion (THD) and intermodulation distortion (IMD), although today the information that used to be wrapped up in these single-figure metrics is often displayed in spectral analyses.

You might infer from the ubiquity of these measurement methods that they are firmly rooted in psychoacoustic reality. Their purpose, after all, is to cast some light on the performance of the device under test when it's used to reproduce music for human ears to appreciate. But this is not the case.

It has been understood for over half a century that THD measurements have poor correlation with perceived distortion, and conventional two-tone IMD measurements are little better. Multi-tone measurements, like that proposed by Alan Belcher, are an improvement but have never become widely used, not least because the measurement of harmonic distortion has a leaden inertia born of its simplicity. It is easy to perform and its results are easy to interpret—that is, until you ask the awkward question: what do they mean for the listening experience?

WEIGHTING SCHEMES

It was to improve the correlation with subjective assessments of distortion while retaining the essential simplicity of measuring equipment using a single, low distortion tone that, down the years, various harmonic weighting schemes were proposed. The idea behind these was simple: that higher-order nonlinearities (which give rise to high-order distortion products in a sine wave measurement) are more aurally objectionable than low-order ones, and should therefore be given greater weight in any subjectively relevant measure of distortion level.

The first such weighting scheme was developed as long ago as 1937 by the Radio Manufacturers' Association of America[1]. It suggested that, in calculating an overall distortion figure, the amplitude of each harmonic should first be multiplied by $n/2$, where n is the harmonic number. The amplitude of the third harmonic is thereby increased by a factor of 1.5, the fourth harmonic by 2, the fifth harmonic by 2.5, and so on. Thirteen years later in 1950, D. E. L. Shorter[2] of the BBC suggested an alternative scheme in which the amplitude of each harmonic was instead multiplied by $n^2/4$, as a result of experiments which showed this to provide better correlation with subjective assessments of distortion level. With this weighting the emphasis applied to higher-order harmonics is even more severe, the third harmonic being increased by a factor of 2.25, the fourth harmonic by 4, the fifth harmonic by 6.25, and so on.

Neither the RMA's nor Shorter's

weighting schemes caught on. One reason for this was that measuring the amplitude of individual harmonics—something which is trivially easy with modern FFT analysis—was exceedingly difficult at the time. Few manufacturers, let alone reviewers, had access to the equipment necessary to achieve this, so harmonic weighting was an idea ahead of its time. A second reason may be that Shorter's weighting was determined empirically, without any justifying theory—an entirely appropriate means by which to determine a psychoacoustically relevant measure of nonlinear distortion, but also an excuse to regard it as unproven.

In fact, as I've shown in a previous article[3], there is theoretical support for Shorter's weighting in that it reflects the amount of intermodulation distortion generated by a given transfer characteristic, and hence gives a much better indication of the total distortion produced on complex signals such as music. (The transfer characteristic, which is central to what follows, is a polynomial equation expressing the output signal in terms of the input signal, and is often depicted as a graph. Some examples are given later.)

In 1961, E. R. Wigan[4], also working at the BBC, came up with a revision of Shorter's weighting in which an overall distortion criterion was calculated using the equation

$$\sum_{2}^{n} n^2 (p_n - t) \text{ percent (for } (p_n - t) > 0)$$

where n is again the harmonic order, pn is the percentage amplitude of the nth harmonic, t is the minimum percentage of the harmonic that is audible, and the Greek sigma indicates that the values for the second to nth harmonics should be summed. Wigan's work confirmed Shorter's belief that the amplitude of each harmonic should be multiplied by a factor proportional to n^2, although the details of his calculation are different. Wigan's weighting scheme also never caught on, one reason for this being that the factor t was not only unknown but unknowable in most instances since it is impossible to generate any order of nonlinearity above the third order in isolation when using a non-steady-state signal like music. So Wigan's criterion was incapable of calculation, even if the amplitudes of individual harmonics were known.

GEDLEE

A new distortion criterion was unveiled at the AES Convention in October 2003, called the GedLee metric as a result of being proposed by Earl Geddes and Lidia Lee[5]. For those who understand mathematical notation, the equation used to calculate it is given in the accompanying sidebar. It differs from the weighting approach adopted by the RMA, Shorter, and Wigan in using the transfer characteristic directly, rather than indirectly via harmonic amplitudes. This, as will become clear, gives it a fundamental advantage.

> ### THE GEDLEE METRIC
>
> The equation for calculating the GedLee metric from a continuous transfer characteristic T(x) is:
>
> $$G_m = \sqrt{\int_{-1}^{1} \left(\cos\left(\frac{x\pi}{2} \right) \right)^2 \left(\frac{d^2}{dx^2} T(x) \right)^2 dx}$$
>
> The first, raised cosine term in the expression weights the metric so that it accords most significance to nonlinearities at low signal amplitudes, close to the zero crossing point. The second term is the second differential of the transfer characteristic, which is a measure of its curvature.

Moreover, the GedLee metric takes into account the fact that the amplitude distribution of wide dynamic range music signals is such that for much of the time the signal amplitude is close to the zero crossing point, with only infrequent excursions to high amplitudes. To account for this, the metric includes a raised cosine weighting function that places greater emphasis on nonlinearity at low signal levels than at high signal levels. The ear also generates greater distortion at high SPLs than at low SPLs, so this weighting is justified by psychoacoustic considerations as well as signal statistics.

These important differences notwithstanding, the GedLee metric is, in a central respect, not entirely novel. The key factor in calculating it is the curvature (second derivative) of the transfer characteristic, and this links it inescapably with Shorter's $n^2/4$ weighting scheme.

As Shorter observed in his 1950 paper: "The importance of high-order harmonics, even when individually small in energy content, suggests that distortion should be regarded in terms not so much of the separate harmonics as of the complete series or of the composite waveform that this series represents. The properties of the waveform that are significant for this purpose remains to be investigated. It may be noted, however, that the product obtained by weighting the distortion waveform proportionately to the square of the frequency, i.e., by adding 12dB per octave rise, or differentiating twice, gives a measure of the reciprocal of the radius of curvature of the waveform, and is therefore related to the sharpness of any corners in it."

This quote clearly shows that Shorter was thinking along essentially the same lines as Geddes and Lee, and that the GedLee metric should be considered a development of his pioneering work. This is confirmed by ***Table 1***, which compares the relative weighting given to each harmonic from the 3rd to 7th by the Shorter method and GedLee metric (assuming in the latter case that only the specified harmonic is generated on a full-scale sine wave

TABLE 1. Comparison of harmonic weighting factors in Shorter's method and the GedLee metric, demonstrating their fundamental similarity.

Harmonic Number	Weighting	
	Shorter	GedLee
3	2.25	2.17
4	4	3.82
5	6.25	5.93
6	9	8.49
7	12.25	11.52

TABLE 2: Harmonic amplitude and phase data used to generate *Fig. 1*.

Harmonic	amplitude (dBr)	polarity
2	−68	−
3	−25	+
4	−62.5	−
5	−32	−
6	−57.5	−
7	−36	+
8	−57.5	−
9	−41	−
10	−58	−
11	−44	+
12	−60	−
13	−47.5	−
14	−60	−
15	−52.5	+
16	−62.5	−
17	−60	−
18	−62.5	−
19	−67.5	+
20	−65	−

input). As you can see, the weighting figures are very similar.

Where the GedLee method scores over Shorter's is that, because it is generated directly from the transfer characteristic, it takes into account the relative phases of the distortion harmonics, which the Shorter weighting does not. An example of where this is of crucial importance is crossover distortion in class B amplifiers. As I've described previously[6], crossover distortion is characterized by a pattern of alternating odd-harmonic polarities which gives rise to the characteristic kink in the transfer characteristic around the zero-crossing point. If, instead, the harmonic polarities are all rendered the same, then the kink disappears and so too does the other characteristic of crossover distortion: an increase in distortion level with decreasing signal amplitude.

This is illustrated by ***Figs. 1*** and ***2***, which show the transfer characteristic (the left curve in each pair) and the transfer characteristic error (right curve) for an example case. These were generated from the data in ***Tables 2*** and ***3***, respectively, which confirm that the amplitude of each harmonic is exactly the same in both instances, the only difference being the polarity of harmonics 3,

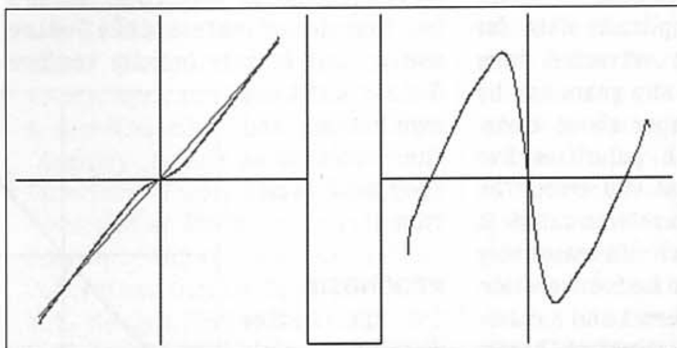

FIGURE 1: Transfer characteristic (red trace, left graph) and transfer characteristic error (right graph) calculated from *Table 2*, representing crossover distortion rated "unacceptable" subjectively. GedLee metric here is 2.22.

7, 11, 15, and 19. A good distortion metric should reflect the difference in transfer characteristic shape and the effect this has on distortion at low signal levels. But the Shorter method, because it is calculated from the harmonic amplitudes, cannot do this: the $n^2/4$-weighted distortion figure is the same in both cases (42.3%!). Contrast this with the GedLee metric (Gm) which is 2.22 for the transfer characteristic of *Fig. 1* and only 0.92 for *Fig. 2*, and so reflects their very different subjective significance.

The harmonic amplitude data for *Figs. 1* and *2* was extracted from spectra published many years ago by James Moir[7] in a paper about crossover distortion; the polarities I've guessed through trial and error. The distortion in this case was rated as "unacceptable," which fits reasonably well with Geddes' and Lee's suggestion that Gm values between 1 and 3 correspond to subjective ratings of "barely perceptible but not annoying," with values less than 1 indicating that the distortion is imperceptible. Moir included two other spectra in his paper corresponding to ratings of "just detectable" and "detectable," for which my reconstructed transfer characteristic and transfer characteristic error graphs are shown in *Figs. 3* and *4*. The Gm values for these are 0.74 and 1.85, respectively, which again accord pretty well with Geddes' and Lee's findings.

Remember that in all these cases the polarities of the harmonics in Moir's spectra have been guesstimated and may not be correct, so this is a less than definitive test of the GedLee metric. But it does broadly confirm Geddes' and Lee's own testing and the conclusions they have drawn from it.

TABLE 3: Harmonic amplitude and phase data used to generate Fig. 2.

Harmonic	amplitude (dBr)	polarity
2	–68	–
3	–25	–
4	–62.5	–
5	–32	–
6	–57.5	–
7	–36	–
8	–57.5	–
9	–41	–
10	–58	–
11	–44	–
12	–60	–
13	–47.5	–
14	–60	–
15	–52.5	–
16	–62.5	–
17	–60	–
18	–62.5	–
19	–67.5	–
20	–65	–

PROGNOSIS

So, the GedLee distortion metric appears to deliver on its promise of offering a simple and effective way of assessing the subjective impact of any audio device's nonlinear distortion. That ought, on the face of things, to ensure it an enthusiastic reception after decades during which the audio industry has failed to get to grips with how to quantify distortion in a perceptually meaningful way. But that protracted failure in itself casts doubts on whether the GedLee metric will ever prove more than a(nother) wasted opportunity.

There is also the practical problem that today's audio measurement tools are generally not equipped to perform the first step in calculating the GedLee metric: synthesis of the device under test's transfer characteristic from its harmonic distortion spectrum. As already described, this requires that the relative phases of the harmonics be determined as well as their amplitudes, something that is readily achievable with FFT spectrum analyzers but not a facility that is routinely provided. It doesn't take much in the way of software elaboration to make good this shortfall, but the fact that anything new is required will, on past evidence, be

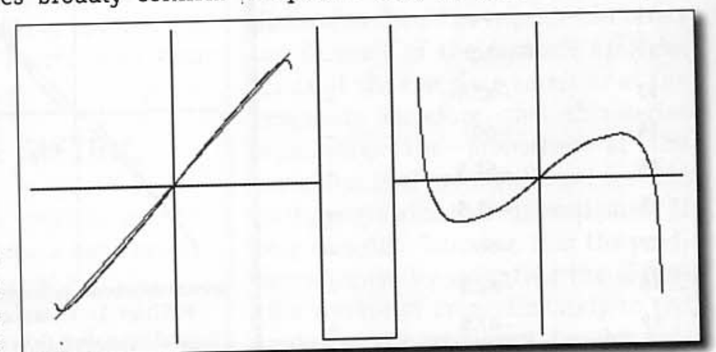

FIGURE 2: Transfer characteristic and error curves calculated from *Table 3*. Although the harmonic amplitudes are identical to *Fig. 1*, the change in four harmonic polarities gives a quite different shape to the curves, reducing the GedLee metric to 0.92.

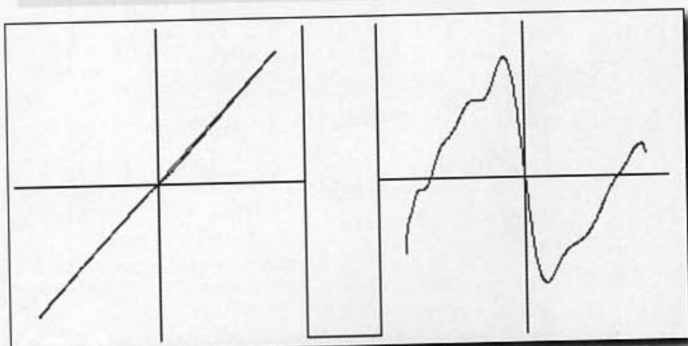

FIGURE 3: As in *Fig. 1* but for less severe crossover distortion rated "just detectable." GedLee metric is 0.74.

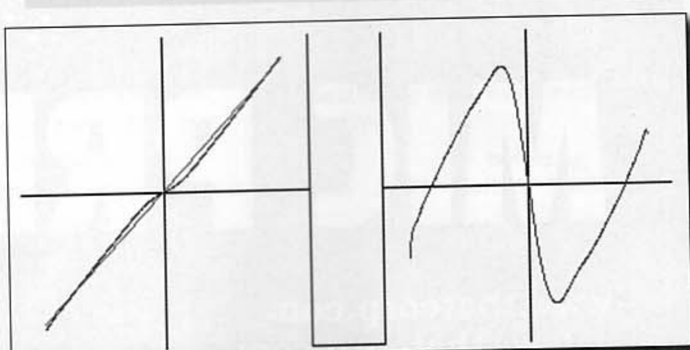

FIGURE 4: As in *Fig. 3* but for a higher level of crossover distortion rated "detectable." GedLee metric is 01.85.

enough to dissuade many from making the effort.

Practical issues also remain to be resolved about how to apply the GedLee metric in circumstances where the transfer characteristic is a function of frequency (as it generally is in both amplifiers and source components) or where the extent of the transfer characteristic that is traversed changes with frequency (as in loudspeakers). Application of the GedLee metric in these circumstances, which together embrace every item of audio equipment, is as yet unclear, and this needs to be clarified before it could enter general and standardized use.

All of which may appear to weigh heavily against the Geddes and Lee proposal ever gaining favor, so here's a rallying cry to all in the audio industry who find the status quo regards nonlinear distortion measurement a lingering stain on our collective escutcheon. Adopt the GedLee metric, use it, refine it, and we may yet consign today's unsatisfactory methods to the dustbin of history, where they have long since belonged. M3

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